

Attenuation of P-Waves by Wave-Induced Fluid Flow

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Attenuation of P-waves by wave-induced fluid flow

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ABSTRACT: Analytical expressions for three P-wave attenuation mechanisms in rocks are given and numerically compared. The mechanisms are: (1) Biot loss, in which flow occurs at the scale of the wavelength between the peaks and troughs of a P wave; (2) squirt loss, in which flow occurs at the grain scale between microcracks in the grains and the adjacent pores; and (3) mesoscopic loss, in which flow occurs at intermediate scales between the various lithological bodies that are present in an averaging volume of earth material. Each mechanism is of importance over different frequency bands. Typically, Biot loss is only important at the highest of ultrasonic frequencies (> 1 MHz), squirt-loss (when it occurs) is important in the range of 10 kHz to 1 MHz, while mesoscale loss dominates at the lower frequencies (< 10 kHz) employed in seismology.

1 INTRODUCTION

In exploration seismology, the material-property resolution required for any forward modeling is dictated by the wavelengths involved and can range anywhere from meters to hundreds of meters depending on the depth of investigation and the seismic source. Averaging elements of earth materials having these dimensions always possess "mesoscopic" heterogeneity in addition to grain-scale heterogeneity. The mesoscopic heterogeneity may be due to variations in lithology (e.g., interbedded sands and shales) or due to macroscopic damage (e.g., highly permeable joints embedded within a much less permeable matrix).

When compressional waves stress such materials, some regions of the material respond with a different fluid pressure than others. The accompanying fluid-pressure diffusion attenuates the wave energy. Three types of wave-induced flow may be identified depending on the distance over which the fluid-pressure must equilibrate.

First, there is the wavelength-scale equilibration between the peaks and troughs of the P-wave. This flow is often called "Biot flow" after the seminal work of Biot (1956). The time necessary for the fluid-pressure differences between the peaks and troughs to equilibrate goes as λ^2/D where λ is the wavelength and D is pore-pressure diffusivity. For P-waves, the wavelength can be estimated as $\lambda = \sqrt{H/\rho}f$,

where H is the undrained compressional-wave modulus (Biot 1962), ρ is the bulk density of the material, and f is frequency. The diffusivity may be written exactly as $D = Mk/\eta$, where M is the fluid storage coefficient defined by Biot (1962), k is the permeability of the material and η is the fluid viscosity. The relaxation frequency f_c , where the Biot-loss per cycle is maximum occurs when the equilibration time equals the wave period

$$f_c = \frac{H}{M} \frac{\eta}{\rho k}, \quad (1)$$

which is also, roughly, the frequency at which viscous-boundary layers develop in the pores of the material [c.f., Eq. (11)]. Thus, at least for Biot loss, the relaxation frequency increases as η/k .

Second, laboratory samples often have broken contacts between the grains and/or microcracks in the grains themselves. Much of this damage occurs as a rock is brought from depth to the surface. Due to diagenetic processes that heal crack damage, it is uncertain whether *in situ* rocks in a sedimentary basin ever have significant numbers of broken grain contacts or open microcracks. Nonetheless, if such grain-scale damage is present, as it is in lab samples at ambient pressures, the fluid-filled microcracks will respond with greater fluid-pressure deviations than the principle porespace resulting in what Mavko and Nur (1979) have called "squirt flow." Perhaps the most

complete description of this loss mechanism is that given by Dvorkin et al. (1995). In squirt flow, the time required for a micro-crack of aperture h and length R (roughly a grain size) to equilibrate with an adjacent pore, goes as $R^2/(h^2 K_f/\eta)$ where K_f is the fluid's bulk modulus. The frequency at which squirt loss is a maximum is therefore

$$f_c = \left(\frac{h}{R}\right)^2 \frac{K_f}{\eta}, \quad (2)$$

which has just the opposite dependence on η as that of the Biot loss, as noted by many authors including Dvorkin et al. (1995). This inverse dependence occurs because the equilibration length R in squirt flow is independent of frequency, while the equilibration length in Biot flow is the wavelength (dependent on frequency).

Finally, there is the "mesoflow" due to the mesoscopic heterogeneity within an averaging volume. More compliant porous materials respond with a greater fluid pressure than do stiffer materials. In the approximation where the mesoscopic heterogeneity can be attributed to just two porous materials, Pride and Berryman (2002) have recently modeled the mesoscopic flow in detail and have obtained general analytical expressions for the frequency dependence of the response. As with squirt flow, the length over which the mesoflow equilibrates is a material property independent of wavelength and so the relaxation frequency is again inversely proportional to the fluid viscosity as in squirt flow.

The goal of the present paper is to give analytical expressions for these three loss processes and to compare them numerically using the unified framework established by Biot.

2 BIOT DESCRIPTION

Biot's (1956; 1962) theory of the linear acoustics of isotropic porous materials is quite general. It accommodates all three of the above loss mechanisms even though Biot himself did not model the detailed frequency dependence of either the squirt or mesoflow mechanisms; however, he did mention these mechanisms.

With an assumed $e^{-i\omega t}$ time dependence, Biot's poroelastic governing equations can be written

$$\nabla \cdot \boldsymbol{\tau} = -\omega^2(\rho \mathbf{u} + \rho_f \mathbf{w}) \quad (3)$$

$$-\nabla p_f = -\omega^2 \rho_f \mathbf{u} - i\omega \frac{\eta}{k(\omega)} \mathbf{w} \quad (4)$$

$$\begin{aligned} \boldsymbol{\tau} = & [(H - 4G/3)\nabla \cdot \mathbf{u} + C\nabla \cdot \mathbf{w}]\mathbf{I} \\ & + G \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T - \frac{2}{3}\nabla \cdot \mathbf{u} \mathbf{I} \right] \end{aligned} \quad (5)$$

$$-p_f = C\nabla \cdot \mathbf{u} + M\nabla \cdot \mathbf{w}. \quad (6)$$

The various fields are: \mathbf{u} , the average displacement of the solid phase within the averaging volume; $-i\omega \mathbf{w}$, the Darcy filtration velocity; p_f , the average fluid pressure; and $\boldsymbol{\tau}$, the average stress tensor acting throughout an averaging volume. The Biot elastic moduli H , C , M and G are, in general, complex and frequency dependent even if Biot (1956, 1962) did not give specific models for such attenuation and dispersion. These constants are exactly related to the constants having the clearest physical interpretation; namely, the drained bulk modulus K_D of the material, the undrained (or "Gassmann") bulk-modulus K_U , and the Skempton's coefficient B that is defined as the ratio of fluid pressure to confining pressure $p_c = -\text{tr}\{\boldsymbol{\tau}\}/3$ under undrained conditions. These general relations are

$$H = K_U + 4G/3, \quad (7)$$

$$C = BK_U, \quad (8)$$

$$M = \frac{B^2}{1 - K_D/K_U} K_U. \quad (9)$$

In the Biot-loss model, K_D , K_U , and B are taken as real constants, while for both the mesoflow and squirt models, the complex frequency dependence of these three constants must be established. In the present paper, the isotropic shear modulus G is taken to be a real constant for all mechanisms since rigorous models of the shear-induced mesoflow and squirt-flow are presently lacking; however, one could use the squirt model of Mavko and Jizba (1991) if an estimate of the frequency dependence of G is desired.

The inertia coefficients are ρ , the bulk-material density, and ρ_f , the fluid density. The coefficient $k(\omega)$ is the complex frequency-dependent dynamic permeability that has been properly modeled by Johnson et al. (1987) as

$$\frac{k(\omega)}{k_o} = \left[\sqrt{1 - i\frac{\omega}{\omega_c} \frac{4}{n}} - i\frac{\omega}{\omega_c} \right]^{-1}, \quad (10)$$

where k_o is the d.c. permeability and where the frequency ω_c corresponds to the situation where viscous-boundary layers begin to form in the pores. Johnson's model predicts that

$$\omega_c = \frac{1}{F} \frac{\eta}{\rho_f k_o}, \quad (11)$$

where F is the electrical formation factor that can be estimated (if desired) from Archie's law $1/F = \phi^m$, where m typically lies in the range $[3/2, 5/2]$. Johnson's model further predicts that $n = \Lambda^2/(k_o F)$, where Λ is a weighted pore-volume to pore-surface ratio; however, as suggested by Johnson et al. (1987),

we simply take $n = 8$ for all materials modeled in this paper.

The P-wave slowness s is determined from the above equations as

$$s^2 = b \mp \sqrt{b^2 - \frac{\rho\tilde{\rho} - \rho_f^2}{MH - C^2}}, \quad (12)$$

where $b = (\rho M + \tilde{\rho} H - 2\rho_f C)/[2(MH - C^2)]$ is simply an auxilliary parameter and where $\tilde{\rho} = -\eta/[i\omega k(\omega)]$ defines the effective inertia of the fluid in relative motion. Taking the minus sign in Eq. (12) gives an s having an imaginary part much smaller than the real part and that thus corresponds to the normal P-wave. Taking the positive sign gives an s with real and imaginary parts of roughly the same amplitude and that thus corresponds to the slow P-wave (essentially, a pure fluid-pressure diffusion). We are only interested here in the normal P-wave.

The above expressions are used to calculate both the P-wave phase velocity v_p and the attenuation measure Q_p^{-1} for all three loss mechanisms; namely,

$$v_p = 1/\text{Re}\{s\} \quad (13)$$

$$Q_p^{-1} = \text{Im}\{s\}/\text{Re}\{s\}. \quad (14)$$

3 MESOSCOPIC FLOW

To estimate the attenuation due to mesoscopic flow, we use the theory of Pride and Berryman (2002) that is rigorously correct whenever the mesoscopic heterogeneity can be attributed to just two porosity types. In such a double-porosity model, the porous material is a composite of two distinct porous continua. Each porous constituent as well as the overall composite is taken to be isotropic; however, no other geometric or material-property constraints are placed on the mixture. The constituents are labeled with the index $i = 1, 2$ and each has their own suite of Biot coefficients. Specifically, we use K_i to denote the drained bulk-modulus of phase i , B_i to denote the Skempton's coefficient, and α_i to denote the Biot and Willis (1957) parameter that is related to the Gassmann modulus K_i^u of phase i as $\alpha_i = (1 - K_i/K_i^u)/B_i$. Each phase has as well its own shear modulus, permeability and formation factor and both phases are saturated by the same fluid. Finally, the volume fraction of each constituent is denoted v_i for $i = 1, 2$, where $v_1 + v_2 = 1$.

Pride and Berryman (2002) have obtained the effective complex constants K_D , K_U and B of the com-

posite in the form

$$\frac{1}{K_D} = a_{11} - \frac{a_{13}^2}{a_{33} + \gamma/i\omega}, \quad (15)$$

$$\frac{1}{K_U} = a_{11} - a_{22} - \frac{\gamma}{i\omega} - \frac{a_{13}^2 + (a_{23} - \gamma/i\omega)^2}{a_{33} + \gamma/i\omega}, \quad (16)$$

$$B = \frac{-a_{12}(a_{33} + \gamma/i\omega) + a_{13}(a_{23} - \gamma/i\omega)}{(a_{22} + \gamma/i\omega)(a_{33} + \gamma/i\omega) - (a_{23} - \gamma/i\omega)^2}, \quad (17)$$

where the geomechanical constants a_{ij} are given by [c.f., Berryman and Pride (2002)]

$$a_{11} = \frac{1}{K_o} \quad (18)$$

$$a_{22} = \frac{v_1\alpha_1}{K_1} \left(\frac{1}{B_1} - \frac{\alpha_1(1 - Q_1)}{1 - K_1/K_2} \right) \quad (19)$$

$$a_{33} = \frac{v_2\alpha_2}{K_2} \left(\frac{1}{B_2} - \frac{\alpha_2(1 - Q_2)}{1 - K_2/K_1} \right) \quad (20)$$

$$a_{12} = -\frac{v_1Q_1}{K_1}\alpha_1 \quad (21)$$

$$a_{13} = -\frac{v_2Q_2}{K_2}\alpha_2 \quad (22)$$

$$a_{23} = -\frac{\alpha_1\alpha_2K_1/K_2}{(1 - K_1/K_2)^2} \left(\frac{1}{K} - \frac{v_1}{K_1} - \frac{v_2}{K_2} \right) \quad (23)$$

Here, $K_o = \lim_{\omega \rightarrow 0} K_D(\omega)$ is the drained bulk modulus of the composite in the low-frequency limit and is the one constant that is not defined in terms of the underlying constituent properties. All dependence of the a_{ij} on the mesoscopic geometry of the constituents and on the underlying shear properties is contained within K_o . The two constants Q_1 and Q_2 are defined

$$v_1Q_1 = \frac{1 - K_2/K_o}{1 - K_2/K_1}, \quad (24)$$

$$v_2Q_2 = \frac{1 - K_1/K_o}{1 - K_1/K_2}. \quad (25)$$

The remaining model parameter is the complex function $\gamma(\omega)$ that has units of inverse viscosity and is given by

$$\gamma(\omega) = \gamma_o \sqrt{1 - i\frac{\omega}{\omega_m}}, \quad (26)$$

where the mesoscopic flow relaxation frequency ω_m is defined by

$$\omega_m = \frac{\eta B_1 K_1}{k_1 \alpha_1} \left[\left(1 + \sqrt{\frac{k_1 B_2 K_2 \alpha_1}{k_2 B_1 K_1 \alpha_2}} \right) \frac{V}{S} \frac{\gamma_o}{\theta} \right]^2, \quad (27)$$

and where γ_o is given by

$$\gamma_o = \frac{v_1 k_1 (Q_1 - B_o/\beta_1 - B_o/B_1)}{\eta L_1^2} \frac{1 - B_o/B_1}{1 - B_o/B_1} \times \left[1 - \frac{\alpha_2}{\alpha_1} \frac{1 - B_o/B_2}{1 - B_o/B_1} \frac{K_1 L_2^2}{K_2 L_1^2} \right]^{-1} \quad (28)$$

Here, $B_o = \lim_{\omega \rightarrow 0} B(\omega)$ is the low-frequency Skempton's coefficient of the composite given by

$$B_o = -\frac{a_{12} + a_{13}}{a_{22} + 2a_{23} + a_{33}}, \quad (29)$$

while β_1 is defined

$$\frac{v_1}{\beta_1} = \frac{v_1 \alpha_1}{K_1} \frac{1 - Q_1}{1/K_1 - 1/K_2} + \frac{v_2 \alpha_2}{K_2} \frac{1 - Q_2}{1/K_2 - 1/K_1} \quad (30)$$

The ratio V/S is the volume of composite divided by the surface area of the interface separating the two constituents within V . The dimensionless parameter θ is given formal definition by Pride and Berryman (2002), but is expected to be close to unity as discussed by the authors. Finally, the lengths L_1 and L_2 define the distance over which the fluid-pressure gradients are defined in each phase when the fluid-pressure equilibration is almost complete. Pride and Berryman (2002) have rigorously defined these lengths but, due to the complexity of the definitions, they are treated here simply as representing a characteristic linear dimension of each phase.

4 SQUIRT FLOW

Here we consider the Dvorkin et al. (1995) squirt-flow model. In this model, the individual grains making up the solid skeleton are taken to be porous. The "grain porosity" is due either to microcracks in the grains themselves or to broken grain contacts that are themselves crack like. Since such crack-like porosity is more compliant than the principle porespace residing between the grains, there develop fluid-pressure differences between the crack porosity of the grains and the principle pores. The fluid-pressure response and subsequent fluid flow inside the porous grains may thus be modeled using Biot theory (poroelasticity).

This general perspective adopted by Dvorkin et al. (1995) is a highly fruitful way of analyzing squirt flow. Two small issues may be raised about the way Dvorkin et al. (1995) develop their specific model: (1) it is unnecessary to assume that the grain space may be divided into effective unit cylinders aligned in the direction of wave propagation and that are immersed in a reservoir of constant fluid pressure; and (2) in taking the limit where the crack porosity is small, these authors assume that the effective drained bulk modulus K_D^g of the porous grains is independent of the

grain porosity ϕ_g which results in a spurious dependence on the fluid modulus K_f . Here, for simplicity, we continue to make the assumption of the first issue; however, we attempt to correct the second issue.

The key part of the Dvorkin et al. (1995) model is to obtain an effective complex solid modulus K_g of the porous grains that replaces the mineral modulus in the standard Biot theory and to obtain the effective complex drained modulus K_D . It is assumed that if the cracked grains are somehow healed without changing the topology of the principle porespace, then a so-called "high-pressure" drained modulus K_D^{hp} may be defined with K_s now representing the true modulus of the single mineral making up the intact grain space. Dvorkin et al. (1995) correctly state that

$$\frac{1}{K_D} - \frac{1}{K_g} = \frac{1}{K_D^{hp}} - \frac{1}{K_s} \quad (31)$$

This equation serves to define K_D under the restriction that all grains are made of a single isotropic mineral. Using the cylindrical-grain assumption referred to above, Dvorkin et al. (1995) determine the effective complex mineral modulus of the porous grains to be

$$K_g(\omega) = \frac{K_D^g + \alpha_g^2 M_g [1 - f(\omega)]}{1 - \alpha_g B f(\omega)}, \quad (32)$$

where K_D^g is the previously mentioned drained-modulus of a porous grain, α_g is the Biot-Willis parameter of a porous grain

$$\alpha_g = 1 - K_D^g/K_s, \quad (33)$$

and B is the Skempton's coefficient of the single-mineral porous material

$$B = \frac{1/K_D^{hp} - 1/K_s}{1/K_D^{hp} - 1/K_s + \phi_p(1/K_f - 1/K_s)}, \quad (34)$$

with ϕ_p the porosity of the principle porespace ($\phi_g \ll \phi_p$). The Biot storage coefficient M_g of the porous grains is defined

$$M_g = \frac{K_f}{\phi_g} \frac{1}{1 + \Delta_g}, \quad (35)$$

where Δ_g is the auxiliary term

$$\Delta_g = \frac{1 - \phi_g}{\phi_g} \frac{K_f}{K_s} \left(1 - \frac{K_D^g}{(1 - \phi_g)K_s} \right). \quad (36)$$

All frequency dependence is confined to the complex function $f(\omega)$ given by

$$f(\omega) = \frac{2J_1(\sqrt{i\omega/\omega_{sq}})}{\sqrt{i\omega/\omega_{sq}} J_0(\sqrt{i\omega/\omega_{sq}})}, \quad (37)$$

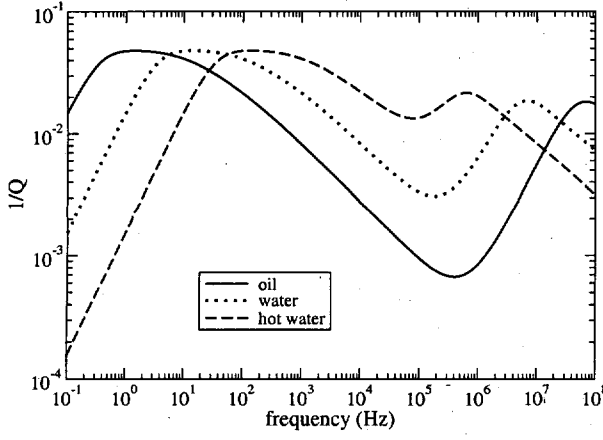


Figure 1: The P-wave attenuation of a double porosity composite having the properties discussed in the text. The three curves correspond to the viscosities of 10^{-2} Pa·s (oil), 10^{-3} Pa·s (ambient water), and 10^{-4} Pa·s (hot water).

where J_0 and J_1 are Bessel functions and where the relaxation frequency is defined by

$$\omega_{sq} = \frac{k_g}{R^2} \frac{M_g}{\eta}. \quad (38)$$

Here, k_g is the permeability of the porous grains and R is a characteristic grain radius.

These choices determine both the effective complex drained modulus $K_D(\omega)$ and the real constant B . It remains only to state the standard Gassmann (1951) result for the undrained modulus

$$K_U(\omega) = K_D(\omega) / [1 - \alpha(\omega)B], \quad (39)$$

where $\alpha(\omega) = 1 - K_D(\omega)/K_g(\omega)$. Using these three expressions for K_U , K_D and B in Eqs. (7)–(9) determines the complex H , C and M of Biot theory, which in turn determines the squirt loss via Eq. (14).

5 NUMERICAL EXAMPLES

Some examples of P-wave attenuation are now given. All examples were determined using Eq. (14) and the appropriate H , C and M for the model.

In the mesoscopic flow model, the drained modulus K_o of the double-porosity composite is taken to be a harmonic average of the two constituent drained moduli, i.e., $K_o = K_1^{v_1} K_2^{v_2}$. Phase 2 in the following examples is assumed to be “embedded” within phase 1 so that $L_1 > L_2$, and we assume for all curves that $L_1 = 2L_2$. Because of such embedding, we also make the crude modeling estimates of $V/S = (L_1 + L_2)^3 / (6L_2^2)$, $v_2 = L_2^3 / (L_1 + L_2)^3$, and $v_1 = 1 - v_2$. These choices formally correspond to phase 2 being little cubes embedded within phase 1. Although the detailed nature of the function $Q^{-1}(\omega)$ changes depending on how L_1/L_2 , V/S , and v_2 are modeled, our present choice is nevertheless representative of the level of attenuation that is present in

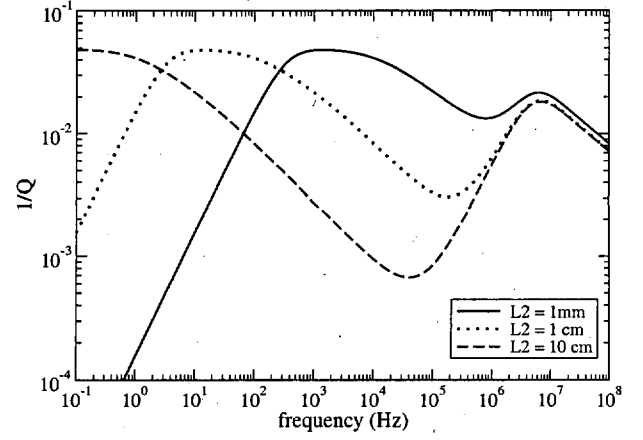


Figure 2: The P-wave attenuation of a double-porosity composite when the size L_2 of the embedded phase varies at fixed ratio $L_1/L_2 = 2$. It is not possible to consider smaller lengths than 1 mm because the notion of a porous continuum requires at least a few tens of grains to be present in each phase.

double-porosity models. In the limited space here, exhaustive examples are not possible.

We assume that the zero-frequency drained moduli in each phase K_i can be determined from the effective-medium formula

$$K_i = \frac{(1 - \phi_i)K_s}{1 + a_i\phi_i}, \quad (40)$$

where the degree of frame consolidation is controlled by the dimensionless factor a_i . For all curves, we assume that phase 1 is a well consolidated sandstone and take $\phi_1 = 0.15$, $a_1 = 10$, and $k_{o1} = 10^{-14}$ m² (where k_{o1} is the d.c. permeability of phase 1). Phase 2 is taken to be a less well-consolidated sand having $\phi_2 = 0.25$, $a_2 = 200$, and $k_{o2} = 10^{-13}$ m². The macroscopic dynamic permeability of the composite is taken to be $k(\omega) = v_1 k_1(\omega)$. Both phases are assumed to be occupied by a single isotropic mineral, so that Skempton’s coefficient B_i of each phase is determined using Eq. (34) [replace K_D^{hp} and ϕ_p by K_i and ϕ_i], while the Biot-Willis parameter of each phase is determined using $\alpha_i = 1 - K_i/K_s$.

In the first example given in Fig. 1, we take $L_2 = 1$ cm and vary the fluid viscosity to show how the two peaks in the attenuation vary. The peak to the left for each curve corresponds to the frequency where the mesoscopic structure just has time to equilibrate in one cycle, while the peak to the right corresponds to the Biot-loss maximum which occurs when the entire wavelength of fluid pressure variation just equilibrates in a cycle. Note that the effect of viscosity is to shift the peaks differently as discussed in the introduction. As expected, increasing fluid viscosity has the effect of shifting the peak in mesoscale loss to lower frequencies, because the fluid-pressure diffusivity is decreasing with viscosity. Because the Biot

attenuation peak is always visible in these plots as the high-frequency peak, we do not give a separate plot for a simple uniform solid-frame porous material.

In the second example given in Fig. 2, we keep the porous continuum properties the same as in the first example but change the size L_2 of the embedded phase while keeping the ratio $L_1/L_2 = 2$ constant and using the fixed viscosity of water. Because the amount of material that must equilibrate is independent of frequency, we see that the peak mesoscale loss shifts as the square of the distance as is expected for any diffusion process. Clearly, if several different porosity types were present, and if each porosity type had a different length, one could expect nearly constant Q behavior to emerge over the seismic band of frequencies ($f < 10^4$ Hz).

To model squirt flow, we need to propose a model for the effective porous grain properties k_g (grain permeability) and K_D^g (drained bulk modulus of a porous grain). Since the porosity within the grains is assumed to be due to microcracks, it is reasonable to take $k_g = \phi_g h^2/12$ as the permeability model, where h is a representative aperture (or gap) of the microcracks. The grain porosity is necessarily on the order of $\phi_g = h/R$, where R is a representative grain radius. Last, the drained modulus of a grain is again taken in the form $K_D^g = (1 - \phi_g)K_s/(1 + a_g \phi_g)$ with $a_g = 200$ (just like phase 2 of the mesoscopic model), which corresponds to the microcracks being rather compliant. It is important to emphasize that Dvorkin et al. (1995) take both k_g and K_D^g as fitting parameters, independent of the grain porosity, which can lead to rather different numerical results than those given here. We feel that the present modeling is more internally consistent and therefore more physical.

In Fig. 3, we vary the aspect ratio h/R for a “high-pressure” frame having a consolidation parameter of $a_{hp} = 10$ and $\phi_p = 0.15$, just like phase 1 of the mesoscopic model. As required by Eq. (38), the peak squirt loss shifts to lower frequency as the square of the aspect ratio and decreases in amplitude in direct proportion to the aspect ratio. At ultrasonic frequencies, squirt loss may dominate all other mechanisms, especially in lab samples for which many broken grain contacts are expected to be present. However, in the seismic band of frequencies, squirt loss is not expected to be the controlling loss mechanism.

To conclude, we underline that what people typically call “Biot loss” is the pressure equilibration occurring between the peaks and troughs of a compressional wave when the porous material is taken to be uniform over the wavelength. Mesoscopic heterogeneity over the scale of a wavelength can be responsible for considerable low-frequency attenuation. By varying the parameters of the double-porosity model, a wide range of $Q^{-1}(\omega)$ behavior can be obtained. At-

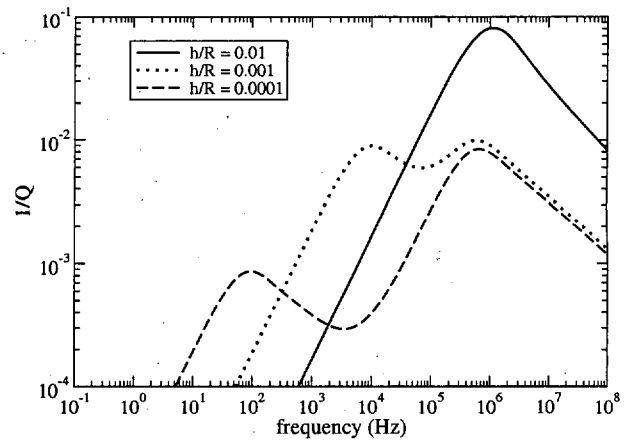


Figure 3: The Dvorkin et al. 1995 squirt model as introduced into Biot theory for three different aspect ratios, where h is an effective micro-crack aperture and R is a grain size.

tenuations can have inverse Q up to $Q^{-1} = 1$, if the embedded phase is taken to be extremely compliant.

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